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Two Time Constant Modeling Approach for Residential Heat Pumps

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TWO TIME CONSTANT MODELING APPROACH FOR RESIDENTIAL HEAT PUMPS*

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ABSTRACT

The paper describes the rationale behind using a two time constant mathematical model to model the indoor coil of a heat pump. The model can be used for either the heating (condenser) or cooling (evaporator) mode. The model is compared to experimental data.

INTRODUCTION

There are many reasons to try to develop a mathematical model of a residential heat pump. Some of these reasons are: (1) for design purposes, (2) for rating purposes, and (3) for control purposes. The complexity of the model needed will be determined by the model usage. If the model is to be used for design purposes a very detailed model is needed, while a model used for rating and control purposes may not need to be as detailed. Even when using models in the design process, if the design of the compressor is being studied, a rather crude model of the heat transfer surfaces (evaporator and condenser) may suffice.

In general the more complex and detailed the model, the bigger the computer, and the longer the computer time needed to solve the resulting model equations. Some very detailed transient models require hours of computer run time to simulate one minute of real time. This is true even using the so-called super computer. Thus, these models are of no value in trying to do active control.

This paper will consider using a two time constant approach to mathematical modeling of the air temperature change across the indoor coil at start up of a residential heat pump. In the cooling mode, the indoor becomes the evaporator. While in the heating mode, the same coil becomes the condenser. Thus, in this paper, the modeling of both the evaporator and the condenser are considered.

The authors do not consider this model as being either detailed or complex. Thus, it will have limited, if any, application to the design of these heat exchangers but it could find application in the controls and rating area.

PREVIOUS MODELS

It would be impossible in the space limits of this paper to attempt to discuss, all mathematical models available in the open literature. A few of the more relevant ones are discussed below.

One of the first transient models of a complete refrigeration system was presented by Dhar [1]. He modeled each heat exchanger as one lump parameter with constant properties. Chi and Didion [2] used an approach similar to Dhar but divided the heat exchanger into several lumps or tanks. Each lump having constant properties.

Chen [3] considered a one-dimensional model of a heat exchanger where all properties could change with both location and time. If Chi and Didion chose enough tanks for the heat exchanger (i.e. the dimension of the tanks become small) their model approaches that of Chen. Chen reported that using a CDC 6500 computer, required 12 hours of computer run time to simulate less than 6 minutes of evaporator real

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time for a start up condition. Although this is the only time reported in the open literature, Chen run times are comparable to that of other people who have developed one dimensional models.

Groff and Bullock [4] first introduced the idea of modeling the temperature change across the indoor coil of a heat pump as a first order system. This idea was further studied and reported by Goldschmidt and Murphy [5,6], Goldschmidt et al. [8] and Bonne et al. [7]. Modeling the coil as a first order system gives:

$$\Delta T = \Delta T_{ss}[1-\exp(-t/\tau)] + \Delta T_{o}\exp(-t/\tau) \qquad (1)$$

where

ΔT = air temperature change across coil
ΔT_{ss} = steady state air temperature change across coil
ΔT_{o} = air temperature change across the coil at time equals zero
t = time
τ = time constant of coil

In most cases $\Delta T_o = 0$ and eq. 1 is non-dimensionalized by dividing by ΔT_{ss}. To give

$$\theta = 1 - \exp(-t/\tau) \qquad (2)$$

when $\theta = \Delta T/\Delta T_{ss}$

The transient capacity of the unit can be obtained by multiplying eq. (1) by the air mass flow rate (m) and specific heat at constant pressure (C_p) and integrating with respect to time.

All of the above authors found the time constant by integrating eq. 2 and comparing it to the area under the measured time-temperature curve. Many people refer to this as "an effective time constant."

Offermann [10] calculated the time constant, τ, by fitting the best experimental curve to the time-temperature difference curve. He also found the effective time constant. He concluded that the coil does not behave as a first order system.

BACKGROUND

Figures 1, 2 and 3 show a normalized time-temperature curve for an evaporator and Figures 5 and 6 show a time-temperature curve for a condenser for the start up conditions. These are typical curves for most air conditioners and heat pumps. These curves have the appearance of an exponential curve. Therefore, it is to be expected that people will try to model the coil as a first order system.

Goldschmidt and Murphy [6] in discussing about what happens to the refrigerant during the compressor off time, gave the reason why the start-up does not behave as a true first order system. They reported that shortly after the compressor turns off, the pressure equalizes inside the heat pump and all components approach their surrounding temperature. The refrigerant pressure always goes to the saturation pressure of the coldest component temperature. In all cases, this is the evaporator. Thus, in all other components the refrigerant will be a superheated vapor. Assuming thermodynamic equilibrium and knowing the total charge of the system, the compressor off time equilibrium mass in each component can be calculated. These calculations show, and later experiments have been verified, that a large percentage of the refrigerant will accumulate in the evaporator and will be in a liquid state. The amount of liquid in the evaporator at the end of the compressor off period is much larger than during steady state running conditions.

When the compressor is first turned on, there are two conditions which control the time required to bring the unit to steady state operations:

1. the time required to bring the metal parts and the refrigerant from compressor off temperature to steady state temperature.
2. the time required to get the excess refrigerant from the evaporator into the rest of the system.

When the heat pump is operating in the heating mode, the indoor coil is the condenser and the outdoor coil is the evaporator. It is still the removal of the refrigerant from the evaporator or outdoor coil that controls the time required to reach steady state operating condition, and thus the steady state temperature change across the indoor coil.

These two conditions lead to the recommendation of considering a model which includes two time constants: one based on the mass of the coil, the second based on the time required to get the excess refrigerant from the evaporator into the rest of the system. When considering the condenser since it is some distance from the evaporator not only a second time constant is needed, there may be a need to use a time delay.

Experimental results [11,12] have shown, that at start up, some of the excess refrigerant inside of the evaporator is pushed out of the system as a liquid and is quickly distributed to the rest of the system. The remainder of the excess refrigerant is boiled off and is redistributed much slower. The refrigerant which is boiled off probably controls the second time constant.

The amount of refrigerant pushed out is a strong function of the coil design and greatly complicates the process of estimating the second time constant.

FORM OF EQUATION

When considering the evaporator, there are two forms the equation could take

$$\theta = 1/2\ [(1-e^{t/\tau_1}) + (1-e^{-t/\tau_2})] \qquad (3)$$

and

$$\theta = (1-e_{-t/\tau_1})\ (1-e^{-t/\tau_2}) \qquad (4)$$

θ = temperature change across coil/steady state temperature change across coil
t = time
τ_1 = time constant based on mass of coil
τ_2 = time constant based on time required to remove excess refrigerant from evaporator.

Equations 3 and 4 both assume that the temperature change across the coil is zero at $t = 0$.

When considering the condenser, both eqs. 3 and 4 may need to be modified by setting

$$t/\tau_2 = 0 \text{ for } t < t_D$$

$$t/\tau_2 = (t-t_D)/\tau_2 \text{ for } t > t_D$$

where t_D = some delay time

DATE

Figure 1 compares eq. 3 for τ_1 = 0.68 minutes and various τ_2 values with experimental evaporator data for a packaged 2 1/2 ton heat pump operating in the cooling mode. τ_2 was obtained by varying the amount of refrigerant "flushed out" at start up. An examination of the data show that a curve between τ_2 = 0.22 and 0.44 minutes would have a reasonable fit to the data.

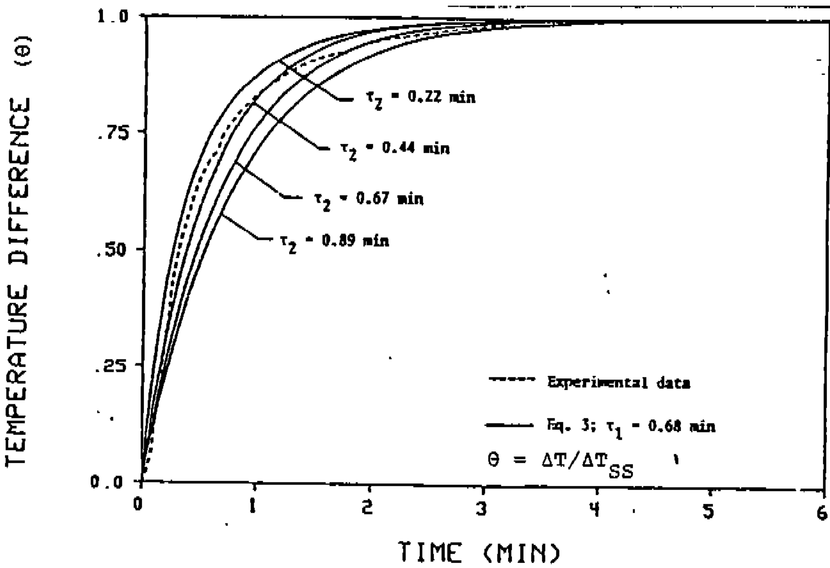

Figure 1. Comparison of two time constant Eq. 3 with experimental data of the indoor coil (evaporator) in cooling mode.

Figure 2 shows the same experimental data compared to eq. 3 with τ_1 = 0.68 and τ_2 = 0.31 minutes. The agreement is excellent. The ratio of the area under the time-temperature experimental curve and the curve obtained using eq. 3 as plotted in Fig. 2 is 1.004. The area under the time-temperature curve is proportional to the unit capacity.

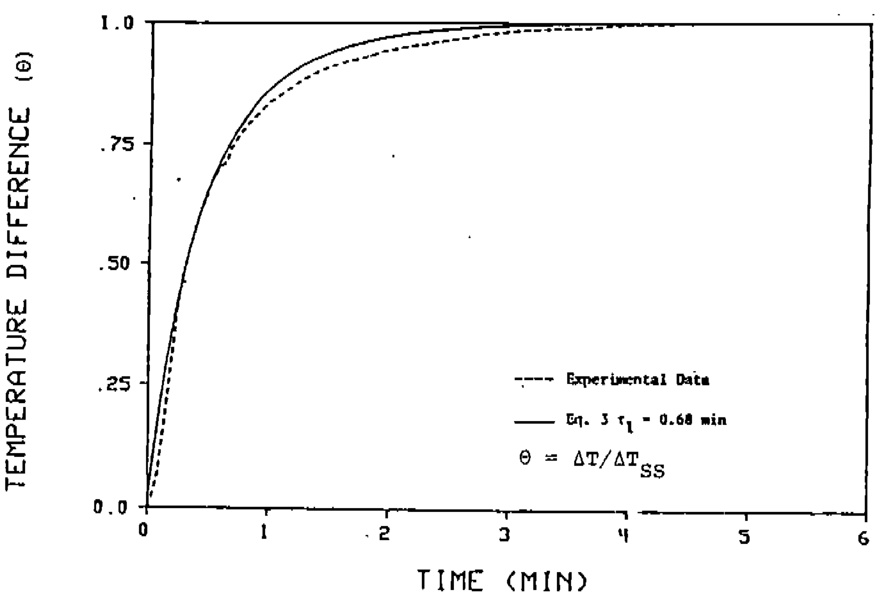

Figure 2. Normalized temperature curve of evaporator compared to Eq. 3 for the two time constant giving best fit.

Figure 3 compares eq. 4 with the evaporator experimental data. The comparison is not good. Mulroy and Didion [11] used a slightly modified form of eq. 4 and compared their equation to experimental data for a split type air conditioner. Their experimental results and comparison with their one and two time constant equations are shown in Fig. 4. Their agreement is excellent. There equation is of the form

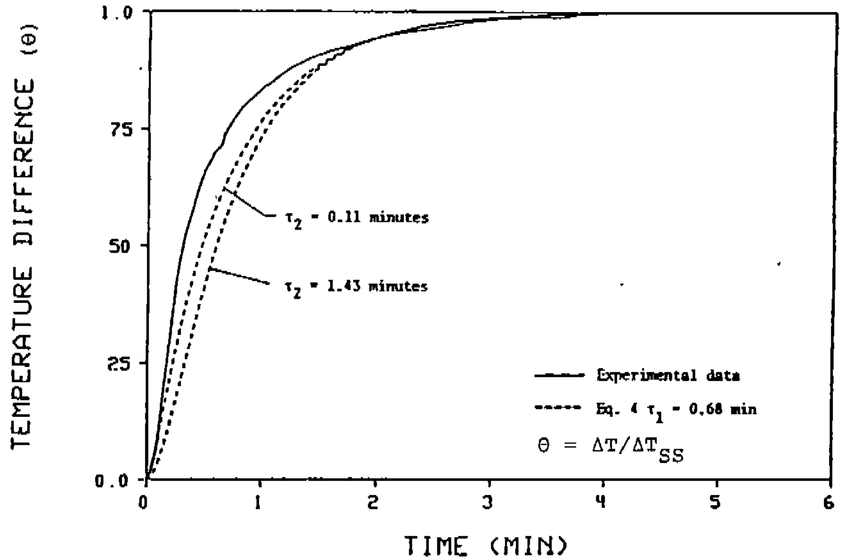

Figure 3. Comparison of two time constant Eq. 4 with experimental pass of the indoor coil (evaporator) in cooling mode.

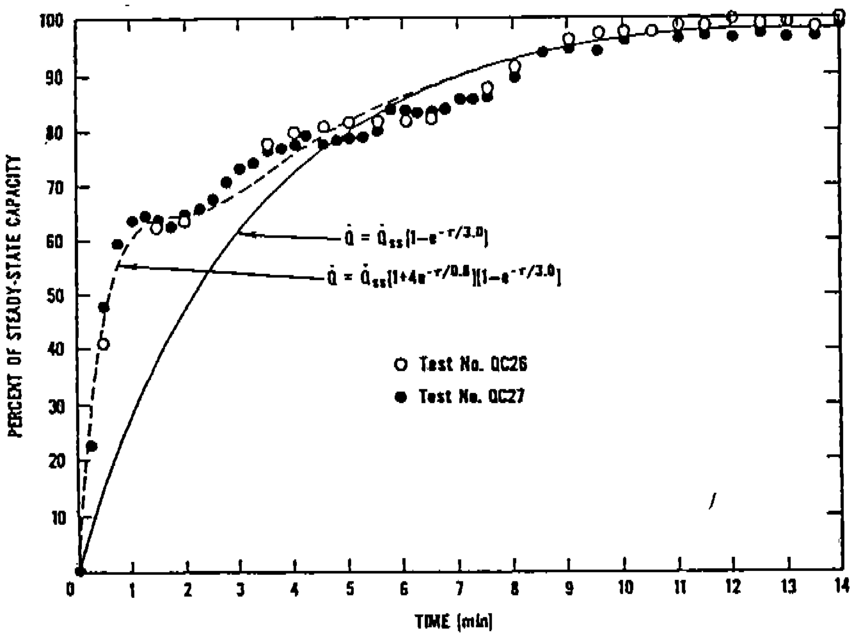

Figure 4. Regressive fit to experimental transient normalized capacity data taken from Reference 10.

$$\theta = (1 + Ae^{-t/\tau_1})(1 + Be^{-t/\tau_2})$$

where A and B are constants. They have added two additional constants. The value of all constants A, B, τ_1, τ_2 are obtained by a regression fit to the experimental data.

Figures 5 and 6 compares eqs. 3 and 4 to experimental data for the condenser, (i.e. indoor coil with heat pump running in heat mode). Neither equations give a very good fit.

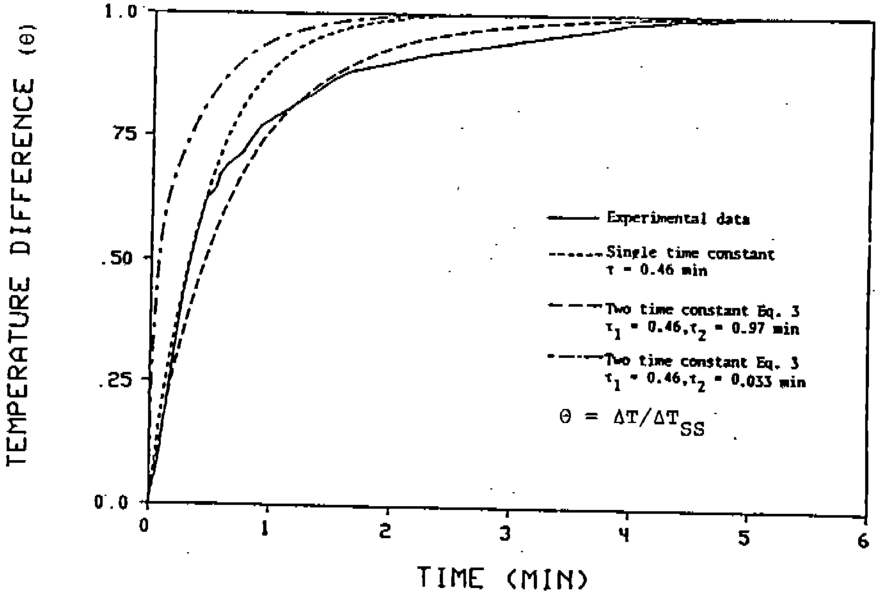

Figure 5. Comparison of two time constant Eq. 3 with experimental data of the indoor coil (condenser) unit operating in the heating mode.

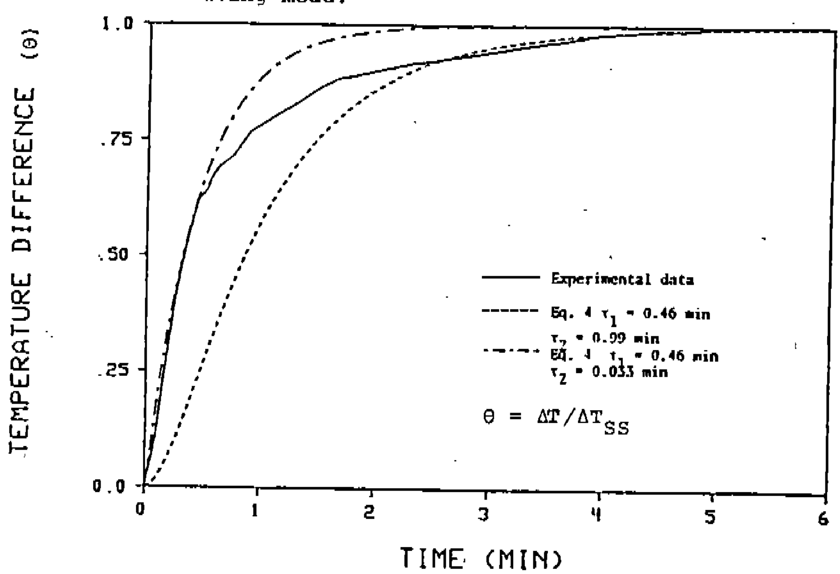

Figure 6. Comparison of two time constant Eq. 4 with experimental data of the indoor coil (condenser) unit operating in the heating mode.

Figure 7 compares eq. 4 with a time delay to the condenser data. The results are much better. A closer look a Fig. 7 would indicate that maybe an additional time constant is needed. There is some physical reasoning why a third time constant should be used.

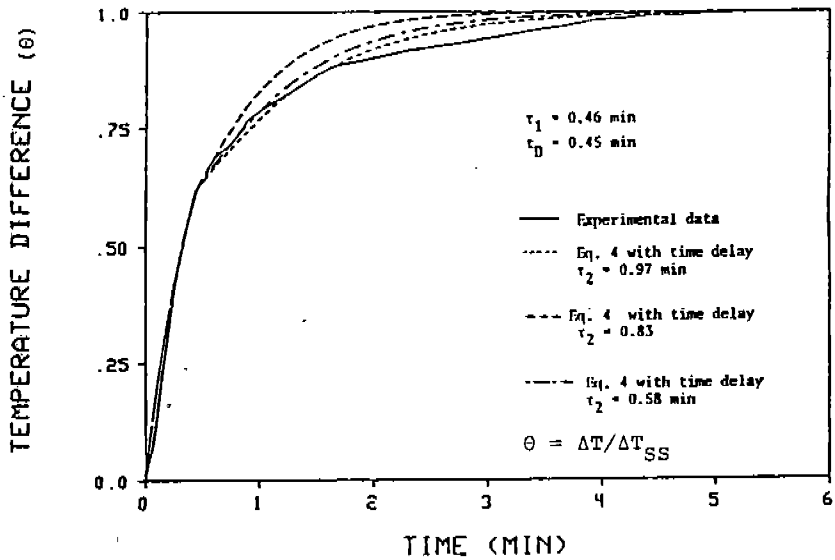

Figure 7. Comparison of Eq. 4 with time delay to experimental data.

The time required to reach steady state is governed by the liquid in the evaporator, and thermal mass of the condenser. But the thermal mass of the evaporator (in this case, the outdoor coil) at any time has a strong influence on the amount of liquid in the evaporator. Thus three time constants based on:

1. thermal mass of outdoor coil,
2. thermal mass of indoor coil, and
3. liquid refrigerant in outdoor coil.

may be needed.

CONCLUSIONS

The results of the work clearly show that a two time constant approach can be used to model the change in air temperature as it flows over the indoor coil of a heat pump both in the heating (condenser) and cooling (evaporator) mode. The area under the temperature - time curve is within 4% of the experimental value for one test and always within 10% for all tests. Since the models requires the determination of two or three constants, these models are well suited to a control system which uses a predictor-corrector approach.

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RÉSUMÉ

Cette étude décrit le raisonnement pour élaborer la spirale intérieure d'une pompe à chaleur, en se servant d'un modèle mathématique constant à deux temps. Le modèle peut être utilisé à la fois pour le chauffage (condensateur) et le refroidissement (vaporisateur). On compare ce modèle avec les résultats des expériences.